Linear combinations of Gamma, (LG) processes and Dirichlet (B-) splines: Applications in finance and insurance

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Abstract

We consider a new class of the so called, LG processes, defined as linear combinations of independent gamma processes. Their distributional and path-wise properties are explored by following their relation to polynomial and Dirichlet (B-) splines. In particular, it is shown that the density of an LG process can be expressed in terms of Dirichlet (B-) splines, introduced independently by Ignatov and Kaishev (1987, 1988, and 1989) and Karlin et al. (1986). We further show that the well known variance-gamma (VG) process, introduced by Madan and Seneta (1990), and the Bilateral Gamma (BG) process, recently considered by Küchler and Tappe (2008) are special cases of an LG process. Following this LG interpretation, we derive new (alternative) expressions for the VG and BG densities and consider their numerical properties. The LG process has two sets of parameters, the B-spline knots and their multiplicities, and offers more flexibility in controlling the shape of the Levy density, than the VG and the BG processes. Such flexibility is often desirable in practice, which makes LG processes interesting for financial and insurance applications.

Multivariate LG processes are also introduced and their relation to multivariate Dirichlet and simplex splines is established. Based on this, expressions for their joint density, the underlying LG-copula and the characteristic function are given. A numerically efficient bridge sampling method for simulating LG sample paths is also developed, based on the Dirichlet bridge sampling of Gamma processes, due to Kaishev and Dimitrova (2009). Estimation of the parameters of LG processes is further considered and a method of moments is developed. Multivariate LG processes are shown to provide a competitive alternative in modelling dependence, compared to the multivariate asymmetric VG process considered by Cont and Tankov (2004) and Luciano and Schoutens (2006), and to its generalization by Luciano and Semeraro (2007) and Semeraro (2008). Applications of these new Dirichlet (B-) spline related processes in finance and insurance, such as modelling the joint dynamics of

multiple exchange rates, valuing exotic options and participating life insurance contracts are also considered.

Keywords. LG process, Variance Gamma process, Bilateral Gamma process, Dirichlet spline, B-spline, simplex spline, Dirichle bridge sampling.

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